

# A Functional Programming Language for Interaction Nets

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Department of Informatics

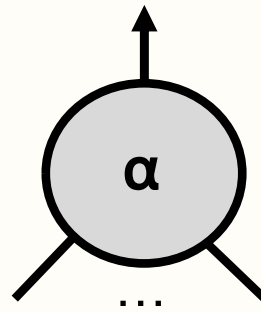
The logo of the University of Sussex, consisting of the letters 'US' in a stylized, serif font.

UNIVERSITY  
OF SUSSEX

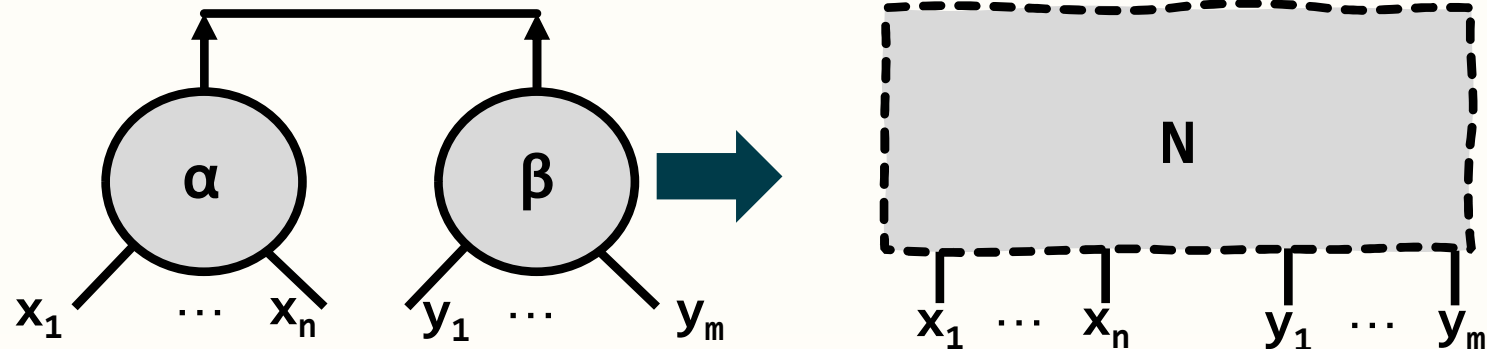
# Interaction nets

Graph rewriting system (Lafont, 1990)

Finite set nodes (“agents”):



Finite set of rewrite rules:



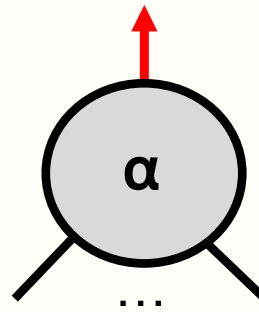
At most one rule per agent pair.

Interface preserved.

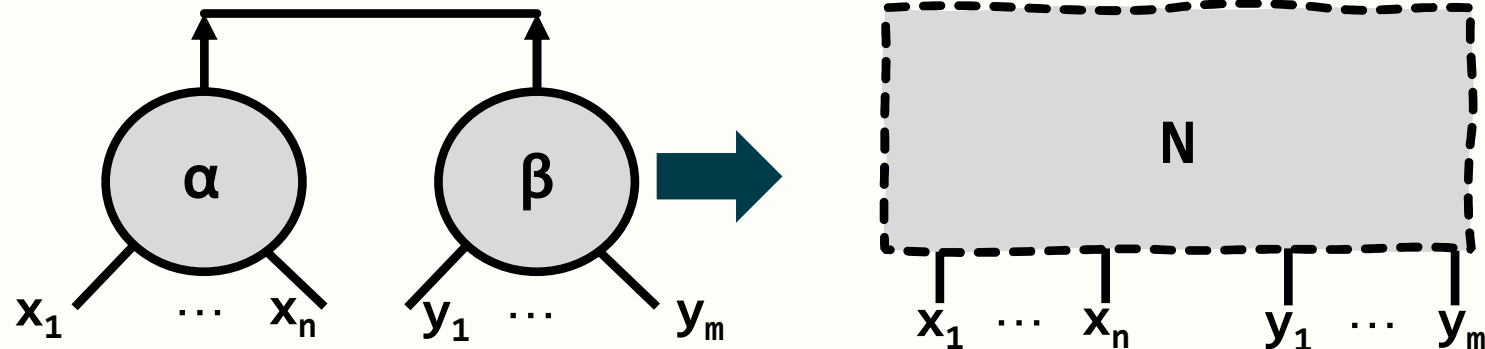
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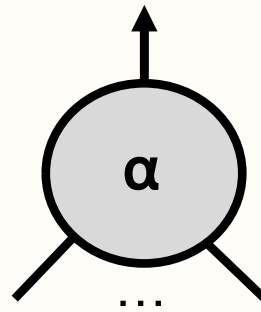
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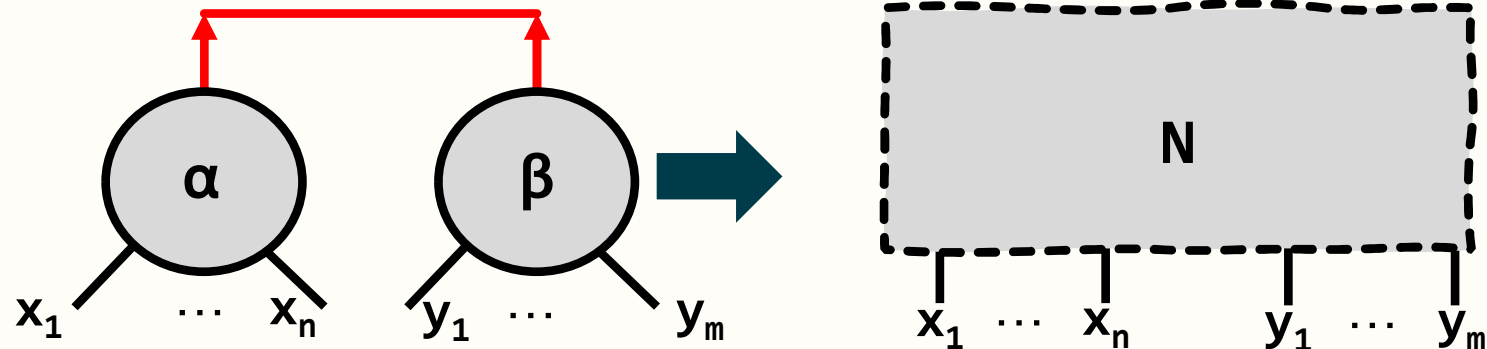
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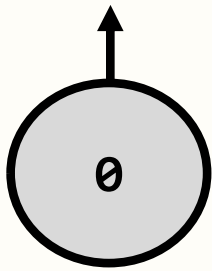


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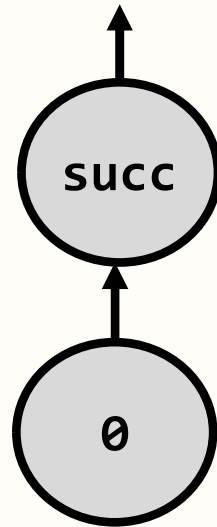
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# Unary numbers

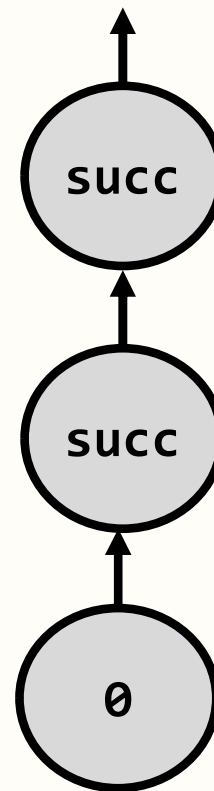
Zero



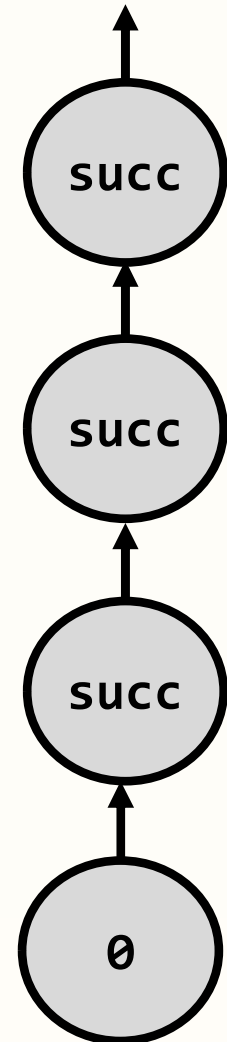
One



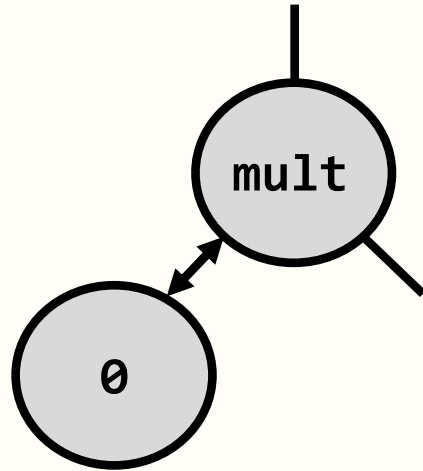
Two



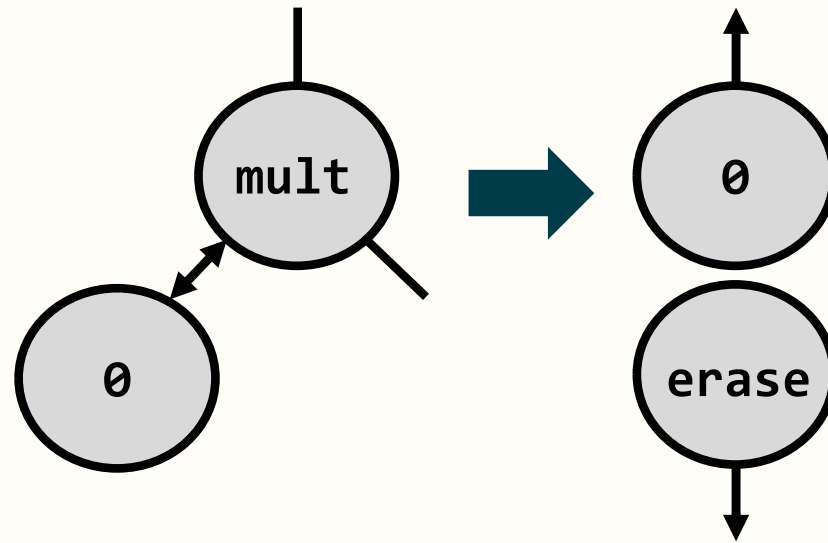
Three



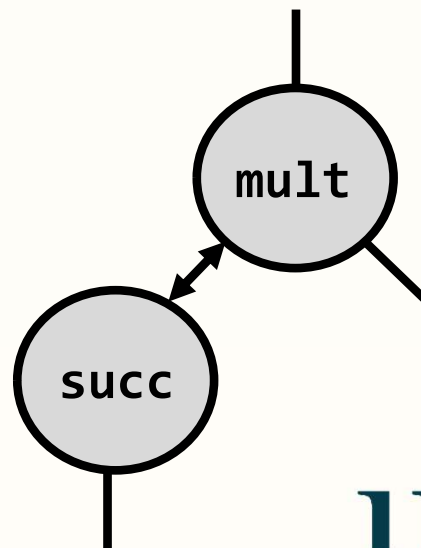
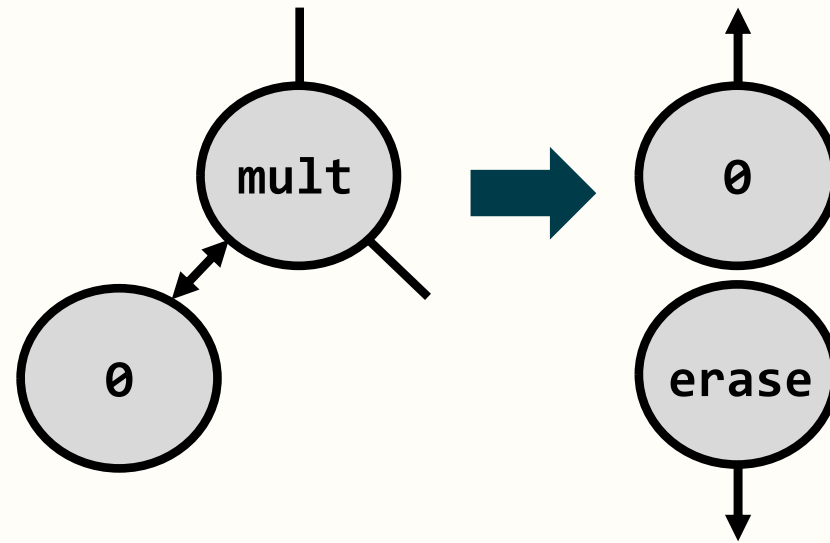
# Unary multiplication



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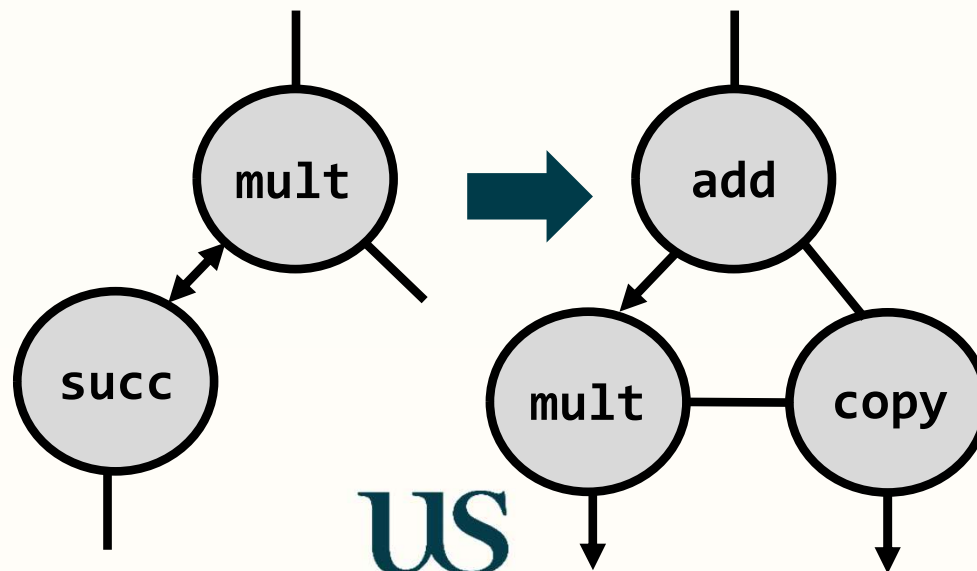
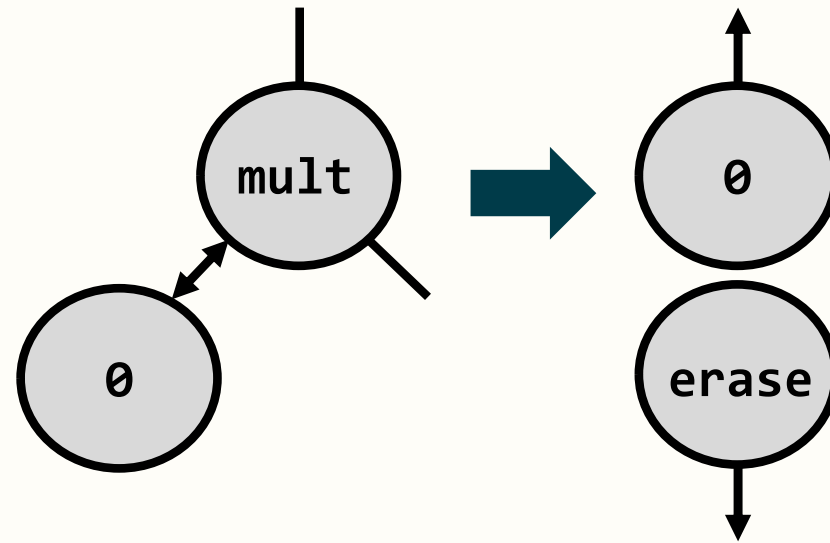


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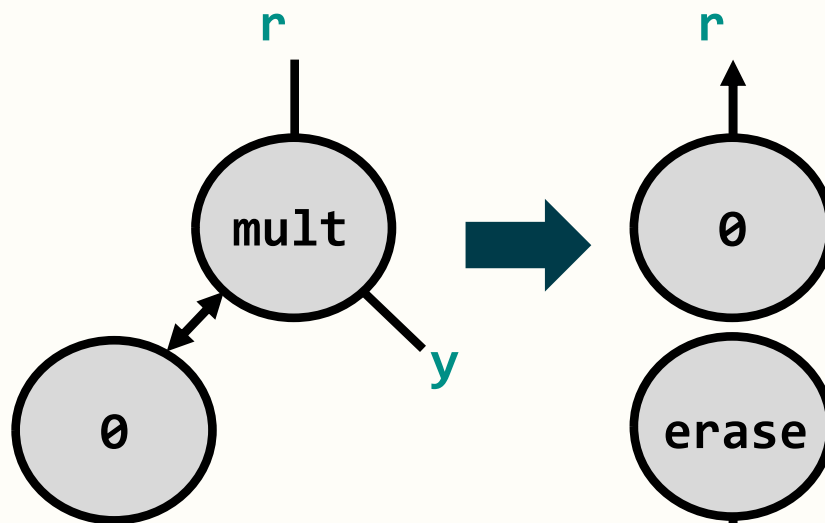


# Unary multiplication



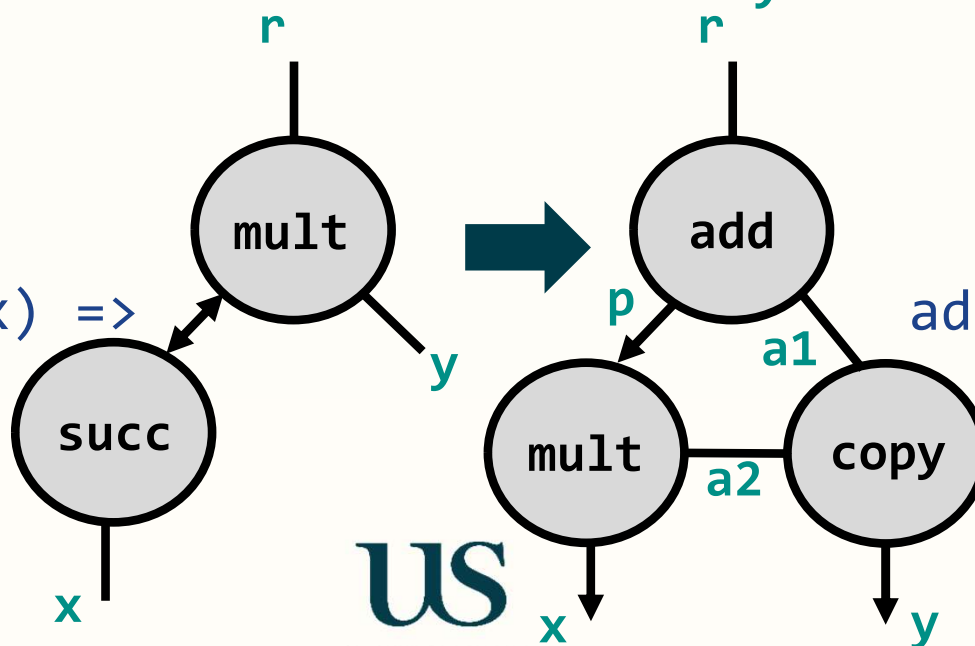
# Explicit textual description

`mult(r,y)><0 =>`



`r~Z, erase~y;`

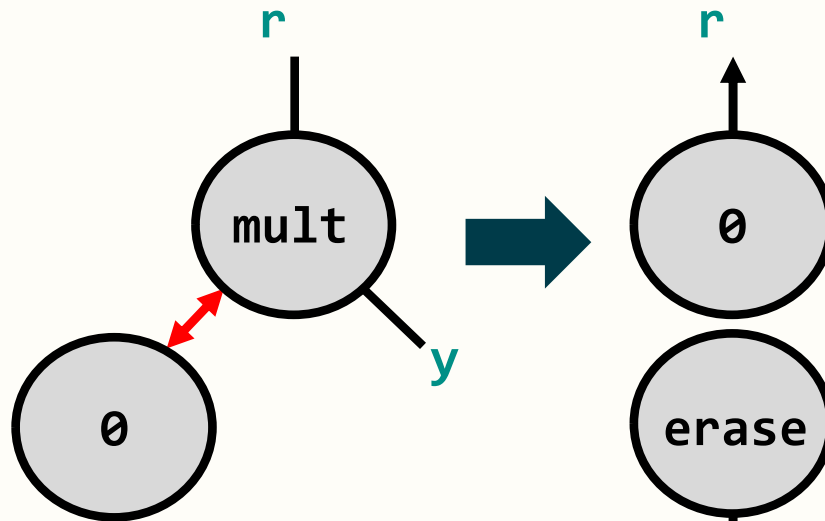
`mult(r,y)><succ(x) =>`



`add(r,a1)~p,  
mult(p,a2)~x,  
copy(a1,a2)~y;`

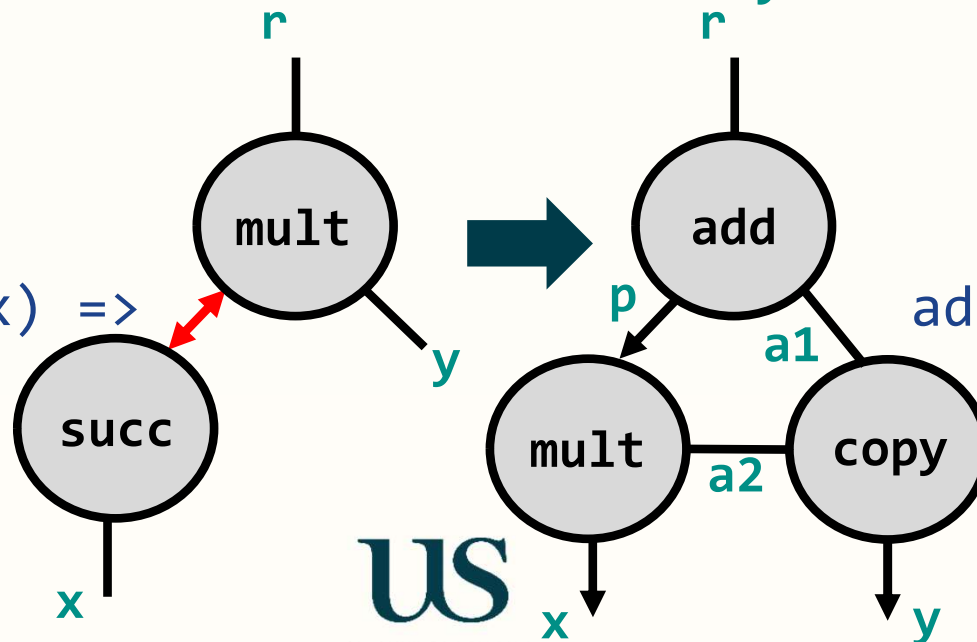
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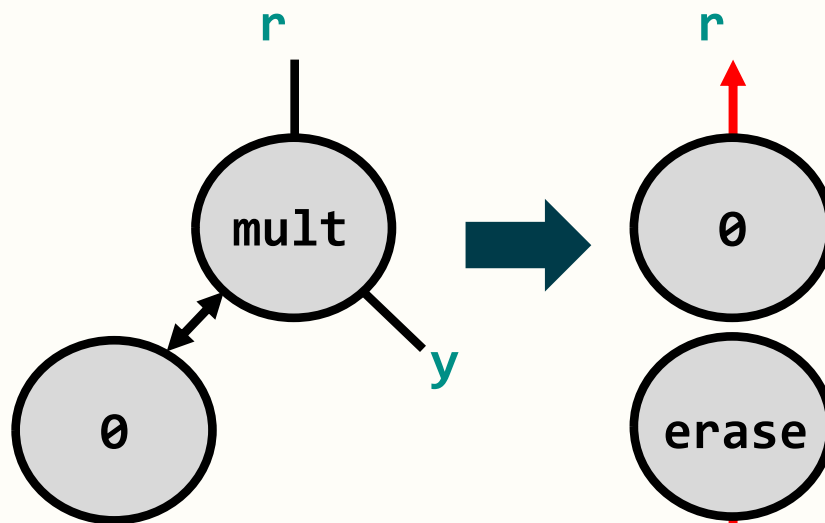
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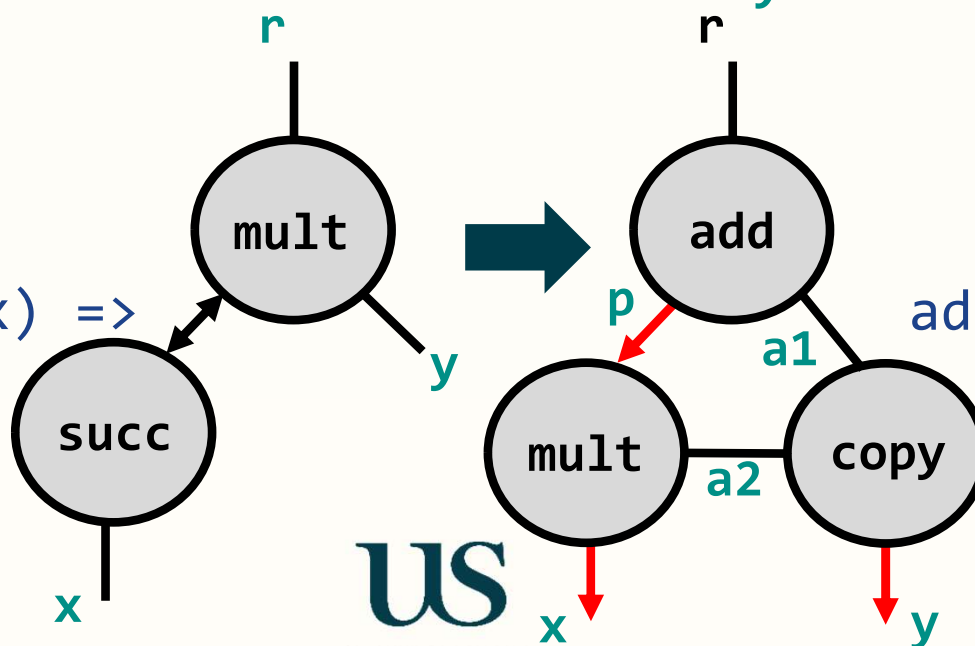
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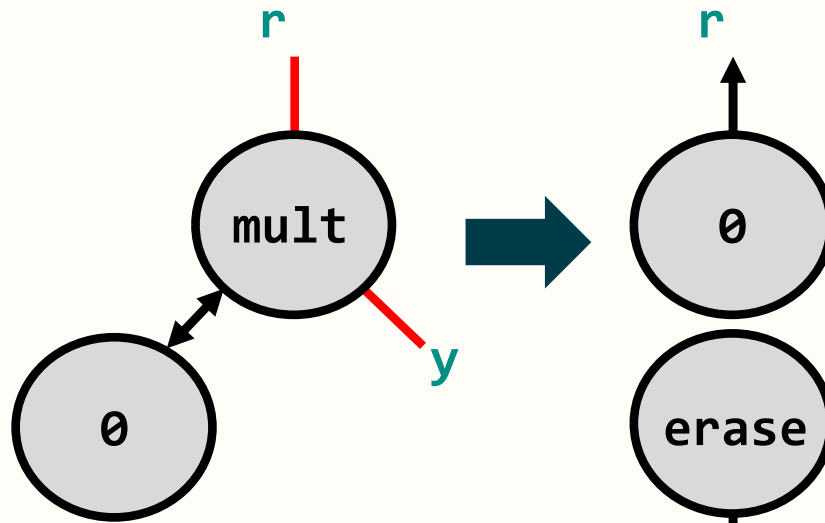
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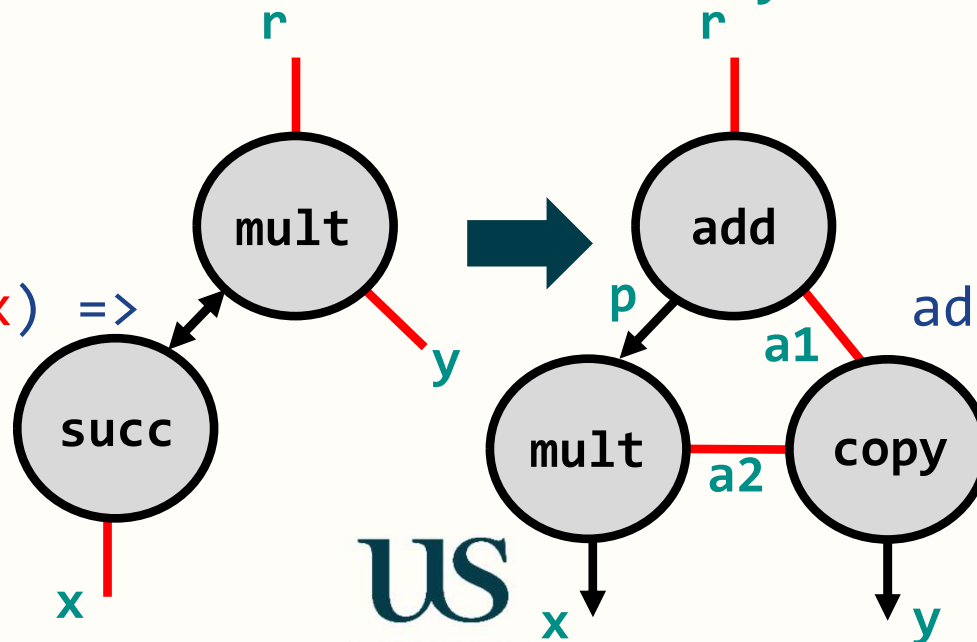
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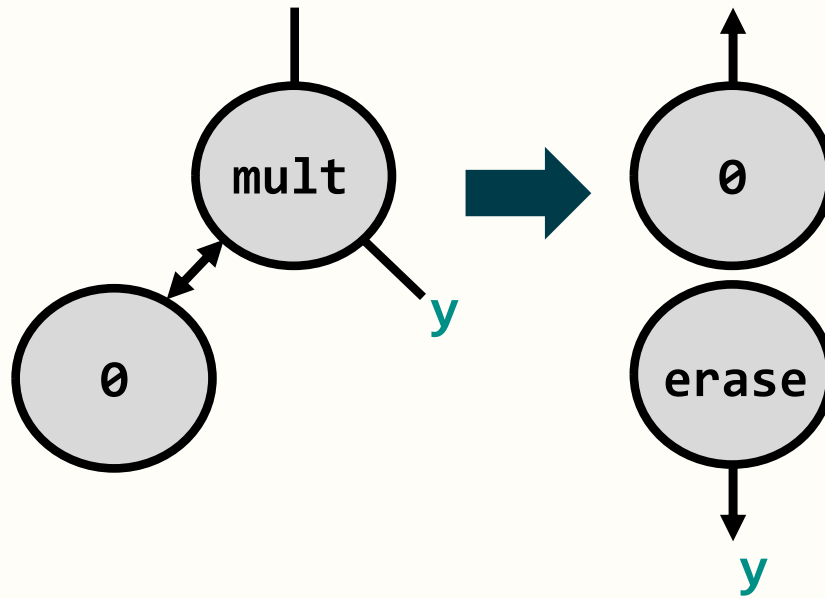
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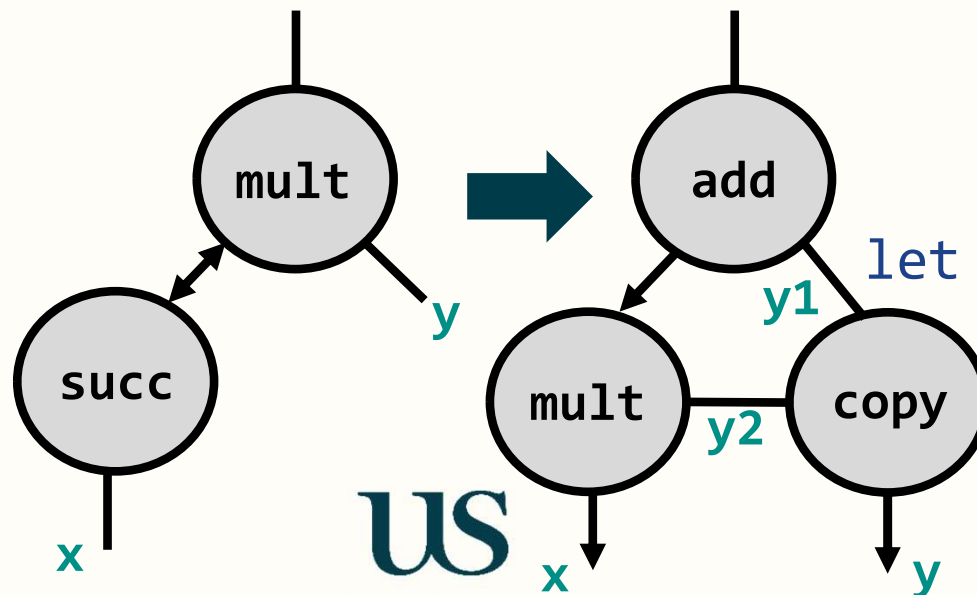
# Functional style description

`mult(0,y)=`



`0 {erase~y}`

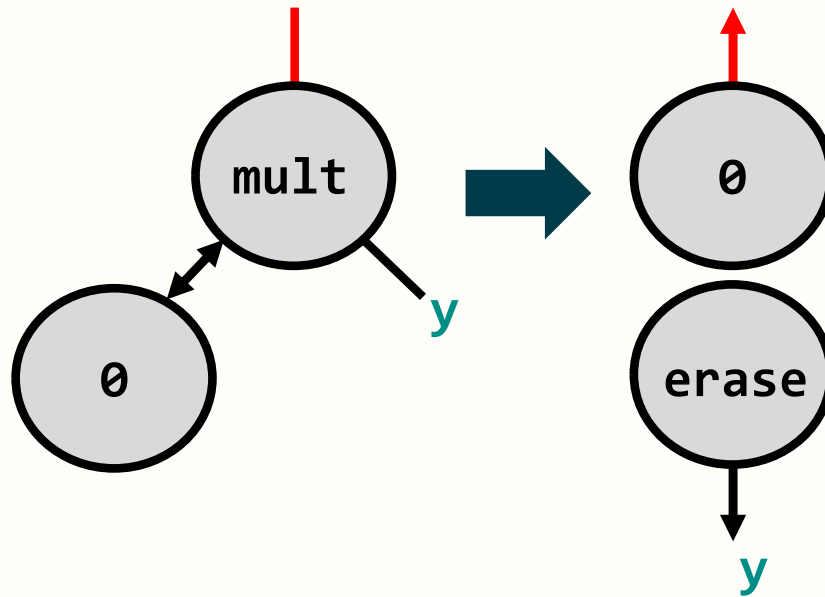
`mult(Succ(x),y)=`



`let <y1,y2>=copy(y)  
in add(  
mult(x,y2),  
y1))`

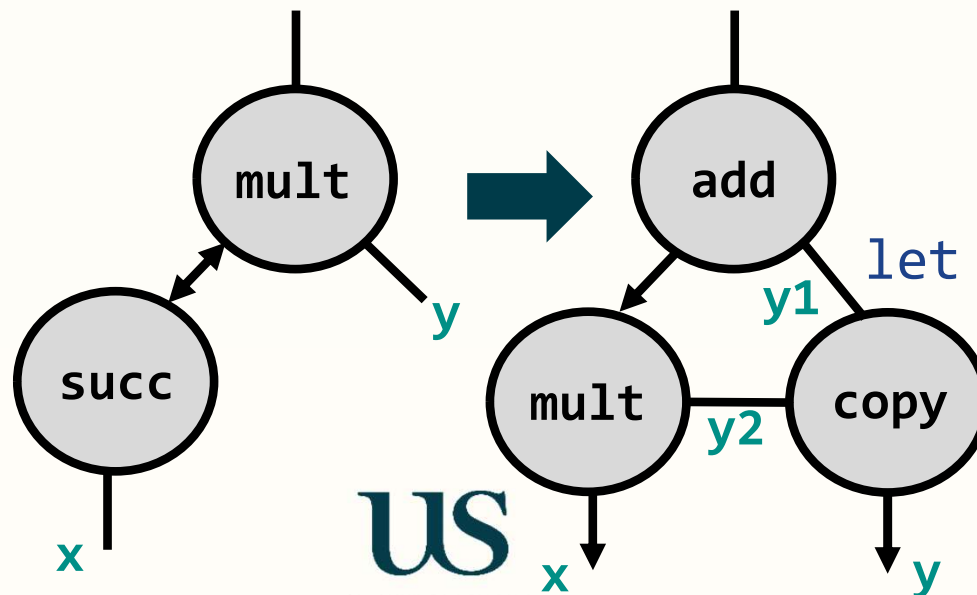
# Functional style description

mult( $\emptyset, y$ ) =



$\emptyset$  {erase~y}

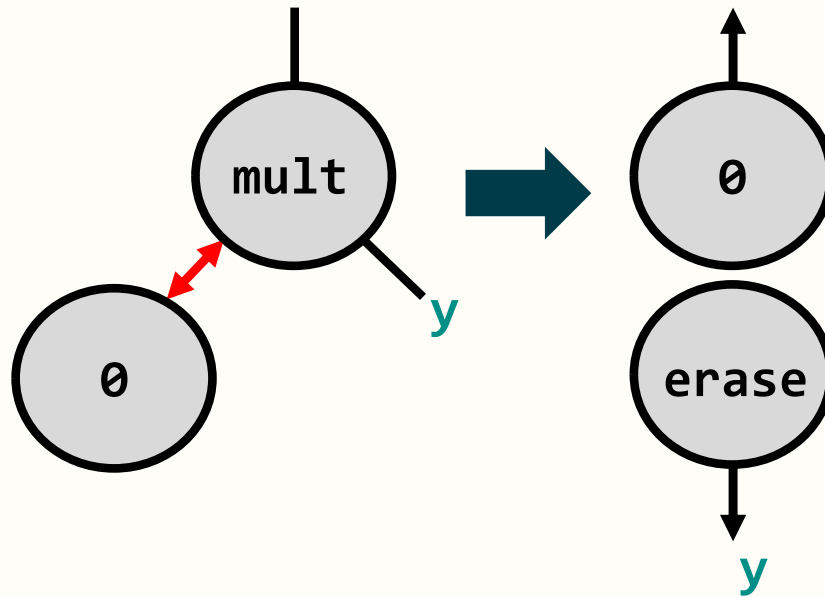
mult(Succ(x), y) =



let  $\langle y1, y2 \rangle = \text{copy}(y)$   
in add(  
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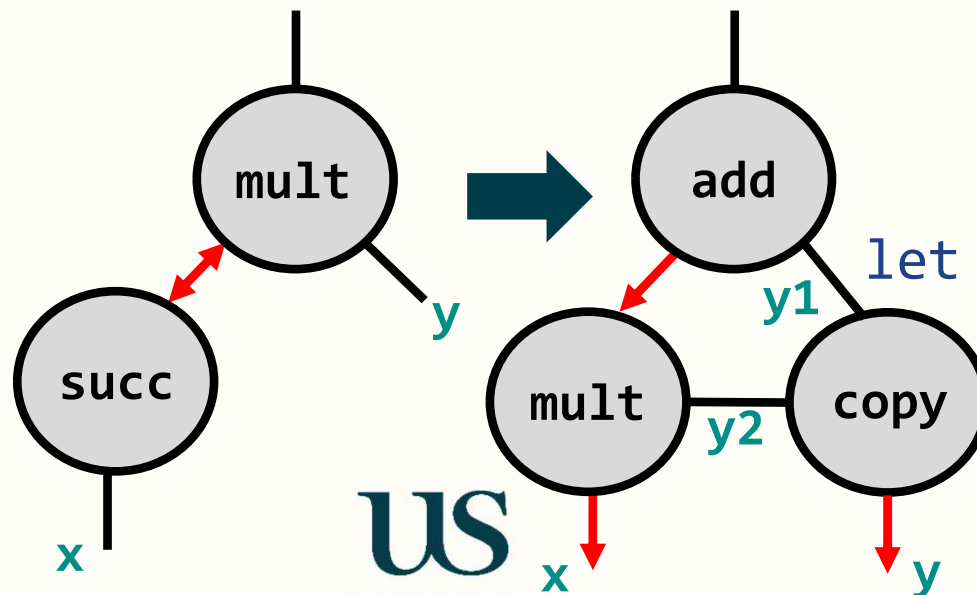
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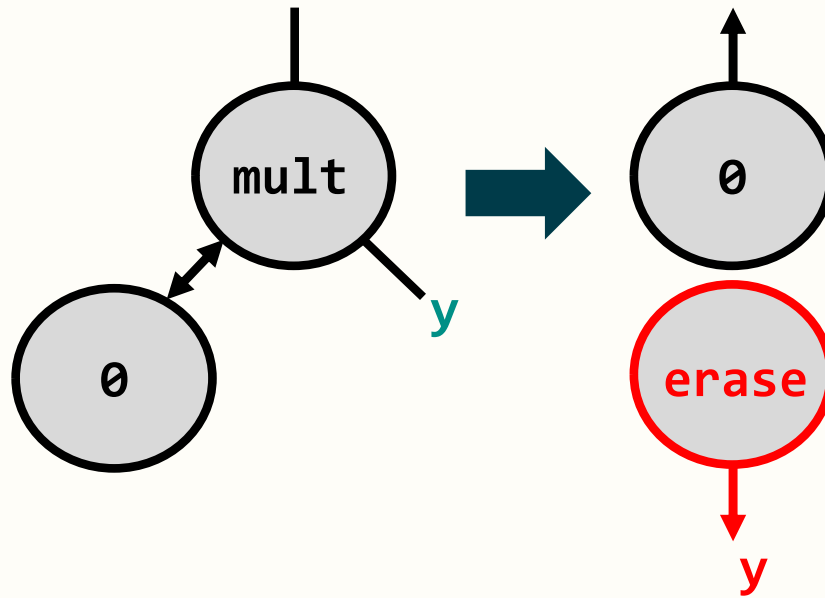


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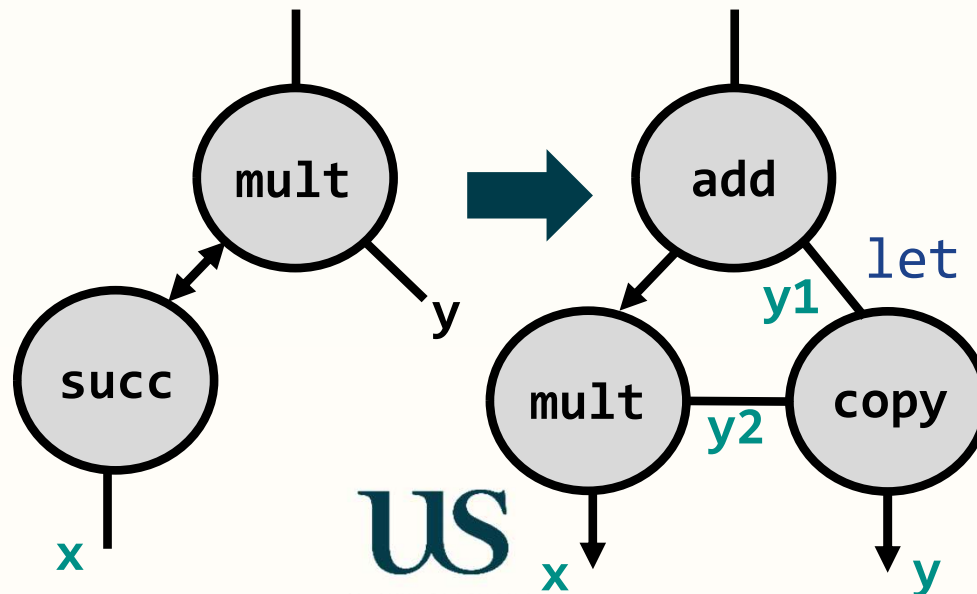
# Functional style description

$\text{mult}(\emptyset, y) =$



$\emptyset \{ \text{erase} \sim y \}$

$\text{mult}(\text{Succ}(x), y) =$



$\text{let } \langle y1, y2 \rangle = \text{copy}(y) \\ \text{in add}( \\ \text{mult}(x, y2), \\ y1))$

# Further examples

$\text{add}(\emptyset, y) = y$   
 $\text{add}(\text{Succ}(x), y) = \text{add}(x, \text{add}(x, y))$

$\text{len}(\text{Nil}) = \emptyset$   
 $\text{len}(\text{Cons}(x, xs)) = \text{Succ}(\text{len}(xs)) \quad \{\text{erase} \sim x\}$

$\text{bubble}(\text{Nil}) = \text{Nil}$   
 $\text{bubble}(\text{Elem } |x|, \text{list}) = \text{bx } |x| (\text{list})$   
 $\text{bx } |x| (\text{Nil}) = \text{Elem } |x| (\text{Nil})$   
 $\text{bx } |x| (\text{list}) = \text{bxy } |x, y| (\text{list})$   
 $\text{bxy } |x, y| (\text{Nil})$   
    |  $x \leq y$                     =  $\text{Elem } |x| (\text{Elem } |y| (\text{Elem}(\text{Nil})))$   
    |  $\_$                              =  $\text{Elem } |y| (\text{Elem } |x| (\text{Elem}(\text{Nil})))$   
 $\text{bxy } |x, y| (\text{list})$   
    |  $x \leq y$                     =  $\text{Elem } |x| (\text{bx } |y| (\text{list}))$   
    |  $\_$                              =  $\text{Elem } |y| (\text{bx } |x| (\text{list}))$

# On-going work

- Lists  $[ ] [Nil, Z]$
- Integer arithmetic  $incr(Num|x|) = Num|x+1|$
- Conditional expressions  $bubble(x:xs) \mid Num(n) = \dots$
- Templates / polymorphism  $copy(\_y) = \langle \_y, \_y \rangle$
- Modules  $import\ sort\ as\ \dots$
- Types  $mult :: nat \times nat \rightarrow nat$

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